Optimal Symbol Detection in the Presence of Interference

Interference-Aware/Unaware Receiver Design

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Outline

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Interference Scenarios

I received a signal $\mathbf{y}$... now what?

Optimal detection depends on what I *know* about $\mathbf{y}$

- System model: narrow/wide-band transmission etc.
- Signal statistics: symbol/noise distribution
- Coherent detection? (Sync, Chest)

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Narrow-band block-fading channel model

$$\mathbf{y}_k = \underbrace{\mathbf{H}_k \mathbf{x}_k}_{\text{useful signal}} + \underbrace{\sum_{j=1, j\neq k}^{K} \mathbf{H}_j \mathbf{x}_j}_{\text{interference}} + \underbrace{\mathbf{n}_k}_{\text{noise}}$$

- $\mathbf{H}_k \in \mathbb{C}^{n_\mathrm{r} \times n_\mathrm{t}}$ channel from eNB to UE k
- $\mathbf{x}_k \in \mathbb{C}^{n_\mathrm{t}}$ transmit signal for UE k, $\mathbf{x}_k = f(\mathbf{s}_k)$, $\mathbf{s}_k \in \mathbb{C}^{n_\mathrm{s}}$
- $\mathbf{n}_k \in \mathcal{NC}(0, \sigma_k^2 \mathbf{I}_{n_\mathrm{r}})$ i.i.d. Gaussian noise vector

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MAP and ML Criterion

Maximize probability of correct detection

- Posterior probability: $P(\mathbf{s}|\mathbf{y}, Z)$, $\mathbf{s} \in \mathcal{Q}_M^{n_\mathrm{s}}$
- Most likely symbol vector $\hat{\mathbf{s}} = \arg\max_{\mathbf{s} \in \mathcal{Q}_M^{n_\mathrm{s}}} \{P(\mathbf{s}|\mathbf{y}, Z)\}$
- Bayes' rule: $P(\mathbf{s}|\mathbf{y}, Z) = \frac{p(\mathbf{y}|\mathbf{s}, Z)P(\mathbf{s})}{p(\mathbf{y}, Z)}$
- $p(\mathbf{y}, Z)$ identical for all symbols $\Rightarrow$ omitted

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Bits are randomized by scrambling/interleaving

- Symbols $\mathbf{s}(i)$ are independent $\Rightarrow$ symbol by symbol detection optimal
- Bits/symbols are approximately uniformly distributed

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MAP $=$ ML if $P(\mathbf{s}) = \frac{1}{Mn_\mathrm{s}}$

- ML criterion: $\hat{\mathbf{s}} = \arg\max_{\mathbf{s}\in\mathcal{Q}_M^{n_\mathrm{s}}}\{f(p(\mathbf{y}|\mathbf{s}, Z))\}$
- Monotonic function $f(x) \Rightarrow f(p(\mathbf{y}|\mathbf{s}, Z))$ likelihood function

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Separate Interference Scenarios

Self-interference (SU-MIMO)

- $\mathbf{y} = \bar{\mathbf{H}}\mathbf{s} + \mathbf{n}$
- Symbols of same UE interfering
- UE knows effective channel $\bar{\mathbf{H}} = \mathbf{HG}$ and constellation $\mathcal{Q}_M^{n_\mathrm{s}}$

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Inter-user interference (MU-MISO)

- $\mathbf{y}_k = \mathbf{H}_k \mathbf{g}_k s_k + \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{g}_j s_j + \mathbf{n}_k$
- Transmissions of different UEs interfering
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Inter-cell interference

- $\mathbf{y}_k = \mathbf{H}_k \mathbf{s}_k + \sum_{j=1, j\neq k}^{K} \mathbf{H}_j \mathbf{s}_j + \mathbf{n}_k$
- Symbols of different cells interfering
- UE may know coarse channel estimates $\hat{\mathbf{H}}_j$

Mixed Interference Scenarios

Multiple streams to multiple UEs (MU-MIMO)

- $\mathbf{y}_k = \mathbf{H}_k \mathbf{G}_k \mathbf{s}_k + \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{G}_j \mathbf{s}_j + \mathbf{n}_k$
- Multiple SU-MIMO transmissions to different UEs interfere
- UE knows effective channels $\bar{\mathbf{H}}_i = \mathbf{H}_k \mathbf{G}_i$, $\mathcal{Q}_{M,k}^{n_s}$ but not $\mathcal{Q}_{M,j}^{n_s}$

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MU-MISO to UE at cell-edge

- $\mathbf{y}_k = \mathbf{H}_k \mathbf{G}_k s_k + \mathbf{H}_k \sum_{j=1, j \neq k}^{K} \mathbf{G}_j s_j + \sum_{l=1, l \neq k, j}^{L} \mathbf{H}_l \mathbf{s}_l + \mathbf{n}_k$
- UE knows effective channels $\bar{\mathbf{H}}_i = \mathbf{H}_i \mathbf{G}_i$ and $\hat{\mathbf{H}}_l$

- SU-MIMO or MU-MIMO transmissions usually not applied for UEs at cell-edge, which are prone to inter-cell interference

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Outline

Single-Transmit Antenna Multiple Receive Antennas

SIMO: $\mathbf{y} = \mathbf{h}s + \mathbf{n}$

- ML: $\hat{s} = \arg\max_{s \in \Omega_M} \{f(p(\mathbf{y}|s, Z))\}$ with $f(p(\mathbf{y}|s, Z)) = \ln p(\mathbf{y}|s, \mathbf{h})$
- $p(\mathbf{y}|s, \mathbf{h}) \in \mathcal{NC}(\mathbf{m}, \mathbf{C})$ with $\mathrm{E}[\mathbf{y}|s, \mathbf{h}] = \mathbf{hs}$ and $\mathbf{C} = \mathrm{E}[\mathbf{nn}^{\mathrm{H}}] = \sigma^2 \mathbf{I}_{n_\mathrm{r}}$
- $\ln p(\mathbf{y}|s, \mathbf{h}) = -\ln(\pi^{n_\mathrm{r}} \sigma^{2n_\mathrm{r}}) - \frac{1}{\sigma^2} \|\mathbf{y} - \mathbf{h}s\|^2$
- $\hat{s} = \arg\min_{s \in \Omega_M} \{\|\mathbf{y} - \mathbf{h}s\|^2\}$

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BICM Soft-decision metric

- Channel decoder require some measure of how likely a certain bit is either 0 or 1 $\Rightarrow$ LLR $\Lambda(\mathbf{y}|s(b_k), \mathbf{h})$
- $\Lambda(\mathbf{y}|s(b_k), \mathbf{h}) = \ln\left(\frac{p(\mathbf{y}|b_k=0)}{p(\mathbf{y}|b_k=1)}\right) = \ln\left(\frac{\sum_{s\in\Omega_M(b_k=0)} \exp\left(-\frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{h}s\|^2\right)}{\sum_{s\in\Omega_M(b_k=1)} \exp\left(-\frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{h}s\|^2\right)}\right)$

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MaxLog Approximation

Sum $\Rightarrow$ Max

- Largest terms in $\exp(x_i)$ tend to dominate for small σ^2
  $\Rightarrow \sum_i \exp(-x_i) \approx \max(\exp(-x_i))$
- $\Lambda(\mathbf{y}|s(b_k), \mathbf{h}) \approx$
  $\max_{s \in \Omega_M(b_k=0)} \left\{-\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{h}s\|^2\right\} - \max_{s \in \Omega_M(b_k=1)} \left\{-\frac{1}{\sigma^2}\|\mathbf{y} - \mathbf{h}s\|^2\right\}$

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- $\Lambda(\mathbf{y}|s(b_k), \mathbf{h}) \approx$
  $\max_{s\in\Omega_M(b_k=0)} \left\{-\frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{h}s\|^2\right\} - \max_{s\in\Omega_M(b_k=1)} \left\{-\frac{1}{\sigma^2}\|\mathbf{y}-\mathbf{h}s\|^2\right\}$

For MaxLog Turbo decoder $\frac{1}{\sigma^2}$ not necessary

$$\begin{aligned}
\lambda &= \max_{s\in\Omega_M} \left\{-\|\mathbf{y}-\mathbf{h}s\|^2\right\} \\
&= \max_{s\in\Omega_M} \left\{-\|\mathbf{y}\|^2 + 2\mathfrak{R}(s^*\bar{y}) - \|\mathbf{h}\|^2|s|^2\right\} \\
&= \max_{s\in\Omega_M} \left\{2\left[\mathfrak{R}(s)\mathfrak{R}(\bar{y}) + \mathfrak{I}(s)\mathfrak{I}(\bar{y})\right] - \|\mathbf{h}\|^2\left[\mathfrak{R}(s)^2 + \mathfrak{I}(s)^2\right]\right\}
\end{aligned}$$

- Matched filter $\bar{y} = \mathbf{h}^{\mathrm{H}}\mathbf{y}$
- Common term $\|\mathbf{y}\|^2$ can be omitted in maximization
- LLR of bit k (b_1 is MSB) given by $\Lambda_k = \lambda(b_k = 0) - \lambda(b_k = 1)$

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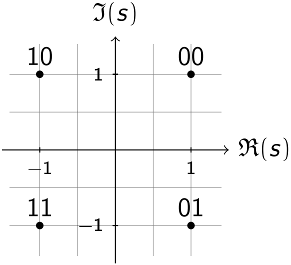
QPSK

Separate LLR into real and imaginary parts

$$\Lambda_k = \max_{b_k=0}\left\{2\Re(\bar{y})\Re(s) - \|\mathbf{h}\|^2\Re(s)^2\right\} + \max_{b_k=0}\left\{2\Im(\bar{y})\Im(s) - \|\mathbf{h}\|^2\Im(s)^2\right\}$$
$$- \max_{b_k=1}\left\{2\Re(\bar{y})\Re(s) - \|\mathbf{h}\|^2\Re(s)^2\right\} - \max_{b_k=1}\left\{2\Im(\bar{y})\Im(s) - \|\mathbf{h}\|^2\Im(s)^2\right\}$$

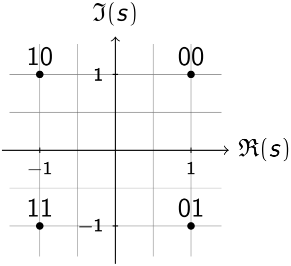


- For $k \in \{1, 3, 5\}$ max over $\Im(s)$ cancel
- For $k \in \{2, 4, 6\}$ max over $\Re(s)$ cancel

QPSK

Separate LLR into real and imaginary parts

$$\Lambda_k = \max_{b_k=0}\left\{2\Re(\bar{y})\Re(s) - \|\mathbf{h}\|^2\Re(s)^2\right\} + \max_{b_k=0}\left\{2\Im(\bar{y})\Im(s) - \|\mathbf{h}\|^2\Im(s)^2\right\}$$
$$- \max_{b_k=1}\left\{2\Re(\bar{y})\Re(s) - \|\mathbf{h}\|^2\Re(s)^2\right\} - \max_{b_k=1}\left\{2\Im(\bar{y})\Im(s) - \|\mathbf{h}\|^2\Im(s)^2\right\}$$



- For $k \in \{1,3,5\}$ max over $\Im(s)$ cancel
- For $k \in \{2,4,6\}$ max over $\Re(s)$ cancel

$$\Lambda_1 = \max_{\Re(s)=+1}\{\Re(\bar{y})\Re(s)\} - \max_{\Re(s)=-1}\{\Re(\bar{y})\Re(s)\}$$
$$= \Re(\bar{y})$$
$$\Lambda_2 = \max_{\Im(s)=+1}\{\Im(\bar{y})\Im(s)\} - \max_{\Im(s)=-1}\{\Im(\bar{y})\Im(s)\}$$
$$= \Im(\bar{y})$$

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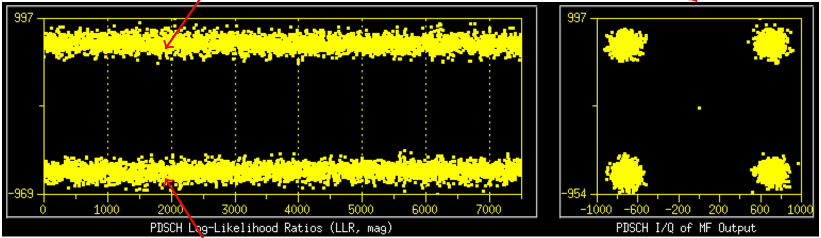
Simulation QPSK

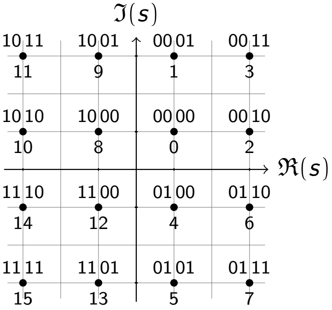
$\Re(\bar{y}), \Im(\bar{y}) > 0$

$\bar{y} = \|\mathbf{h}\|^2 s + \tilde{n}$

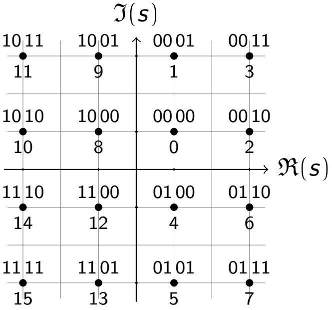


$\Re(\bar{y}), \Im(\bar{y}) < 0$

16-QAM


$\Im(s)$
$\Re(s)$
10 11
11
10 01
9
00 01
1
00 11
3
10 10
10
10 00
8
00 00
0
00 10
2
11 10
14
11 00
12
01 00
4
01 10
6
11 11
15
11 01
13
01 01
5
01 11
7

16-QAM



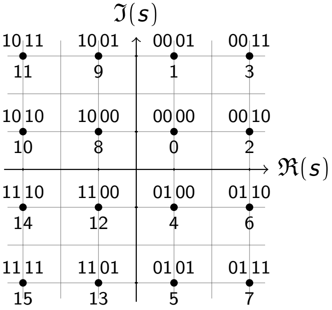
$$\Lambda_1 = \begin{cases} \frac{8}{\sqrt{10}}\Re(\bar{y}) + \frac{8}{10}\|\mathbf{h}\|^2 & \text{if} \quad \Re(\bar{y}) < -\frac{2\|\mathbf{h}\|^2}{\sqrt{10}} \\ \frac{4}{\sqrt{10}}\Re(\bar{y}) & \text{if} \quad |\Re(\bar{y})| \leqslant \frac{2\|\mathbf{h}\|^2}{\sqrt{10}} \\ \frac{8}{\sqrt{10}}\Re(\bar{y}) - \frac{8}{10}\|\mathbf{h}\|^2 & \text{if} \quad \Re(\bar{y}) > \frac{2\|\mathbf{h}\|^2}{\sqrt{10}} \end{cases}$$

$$\Lambda_2 = \begin{cases} \frac{8}{\sqrt{10}}\Im(\bar{y}) + \frac{8}{10}\|\mathbf{h}\|^2 & \text{if} \quad \Im(\bar{y}) < -\frac{2\|\mathbf{h}\|^2}{\sqrt{10}} \\ \frac{4}{\sqrt{10}}\Im(\bar{y}) & \text{if} \quad |\Im(\bar{y})| \leqslant \frac{2\|\mathbf{h}\|^2}{\sqrt{10}} \\ \frac{8}{\sqrt{10}}\Im(\bar{y}) - \frac{8}{10}\|\mathbf{h}\|^2 & \text{if} \quad \Im(\bar{y}) > \frac{2\|\mathbf{h}\|^2}{\sqrt{10}} \end{cases}$$

$$\Lambda_3 = \frac{-4}{\sqrt{10}}|\Re(\bar{y})| + \frac{8}{10}\|\mathbf{h}\|^2$$

$$\Lambda_4 = \frac{-4}{\sqrt{10}}|\Im(\bar{y})| + \frac{8}{10}\|\mathbf{h}\|^2$$

16-QAM



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$$\Lambda_4 = \frac{-4}{\sqrt{10}}|\Im(\bar{y})| + \frac{8}{10}\|\mathbf{h}\|^2$$

- Scale LLRs by $\frac{4}{\sqrt{10}}$ and approximate most significant bits

$$\Lambda_1 \approx \Re(\bar{y}) \qquad \Lambda_3 = -|\Re(\bar{y})| + \frac{2}{\sqrt{10}}\|\mathbf{h}\|^2$$

$$\Lambda_2 \approx \Im(\bar{y}) \qquad \Lambda_4 = -|\Im(\bar{y})| + \frac{2}{\sqrt{10}}\|\mathbf{h}\|^2$$

Simulation 16-QAM

$$\Lambda_1 \approx \Re(\bar{y}) \qquad \Lambda_3 = -|\Re(\bar{y})| + \frac{2}{\sqrt{10}}\|\mathbf{h}\|^2$$
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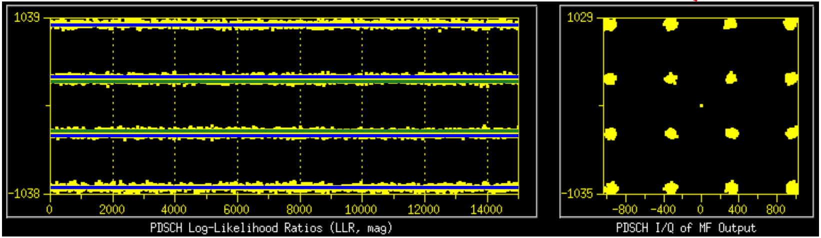
$$\bar{y} = \|\mathbf{h}\|^2 s + \tilde{n}$$



- 75% of LLRs are ± 1 and 25% are ± 3

64-QAM

$$\Lambda_1 = \begin{cases} \frac{16}{\sqrt{42}}\Re(\bar{y}) + \frac{48}{42}\|\mathbf{h}\|^2 & \text{if} \quad \Re(\bar{y}) < \frac{-6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{12}{\sqrt{42}}\Re(\bar{y}) + \frac{24}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{-6\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{-4\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{8}{\sqrt{42}}\Re(\bar{y}) + \frac{8}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{-4\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{-2\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{4}{\sqrt{42}}\Re(\bar{y}) & \text{if} \quad \frac{-2\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{8}{\sqrt{42}}\Re(\bar{y}) - \frac{8}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{4\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{12}{\sqrt{42}}\Re(\bar{y}) - \frac{24}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{4\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) \leqslant \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{16}{\sqrt{42}}\Re(\bar{y}) - \frac{48}{42}\|\mathbf{h}\|^2 & \text{if} \quad \Re(\bar{y}) > \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \end{cases}$$

$$\Lambda_2 = \Lambda_1(\Re \rightarrow \Im)$$

$$\Lambda_3 = \begin{cases} \frac{-8}{\sqrt{42}}|\Re(\bar{y})| + \frac{40}{42}\|\mathbf{h}\|^2 & \text{if} \quad |\Re(\bar{y})| \geqslant \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{-4}{\sqrt{42}}|\Re(\bar{y})| + \frac{16}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant |\Re(\bar{y})| < \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{-8}{\sqrt{42}}|\Re(\bar{y})| + \frac{24}{42}\|\mathbf{h}\|^2 & \text{if} \quad |\Re(\bar{y})| < \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \end{cases}$$

$$\Lambda_4 = \Lambda_3(\Re \rightarrow \Im)$$

$$\Lambda_5 = -\left| -\frac{4}{\sqrt{42}}|\Re(\bar{y})| + \frac{16\|\mathbf{h}\|^2}{42} \right| + \frac{8\|\mathbf{h}\|^2}{42}$$

$$\Lambda_6 = \Lambda_5(\Re \rightarrow \Im)$$

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$$\Lambda_1 = \begin{cases} \frac{16}{\sqrt{42}}\Re(\bar{y}) + \frac{48}{42}\|\mathbf{h}\|^2 & \text{if} \quad \Re(\bar{y}) < \frac{-6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{12}{\sqrt{42}}\Re(\bar{y}) + \frac{24}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{-6\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{-4\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{8}{\sqrt{42}}\Re(\bar{y}) + \frac{8}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{-4\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{-2\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{4}{\sqrt{42}}\Re(\bar{y}) & \text{if} \quad \frac{-2\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{8}{\sqrt{42}}\Re(\bar{y}) - \frac{8}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) < \frac{4\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{12}{\sqrt{42}}\Re(\bar{y}) - \frac{24}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{4\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant \Re(\bar{y}) \leqslant \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{16}{\sqrt{42}}\Re(\bar{y}) - \frac{48}{42}\|\mathbf{h}\|^2 & \text{if} \quad \Re(\bar{y}) > \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \end{cases}$$

$$\Lambda_2 = \Lambda_1(\Re \to \Im)$$

$$\Lambda_3 = \begin{cases} \frac{-8}{\sqrt{42}}|\Re(\bar{y})| + \frac{40}{42}\|\mathbf{h}\|^2 & \text{if} \quad |\Re(\bar{y})| \geqslant \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{-4}{\sqrt{42}}|\Re(\bar{y})| + \frac{16}{42}\|\mathbf{h}\|^2 & \text{if} \quad \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \leqslant |\Re(\bar{y})| < \frac{6\|\mathbf{h}\|^2}{\sqrt{42}} \\ \frac{-8}{\sqrt{42}}|\Re(\bar{y})| + \frac{24}{42}\|\mathbf{h}\|^2 & \text{if} \quad |\Re(\bar{y})| < \frac{2\|\mathbf{h}\|^2}{\sqrt{42}} \end{cases}$$

$$\Lambda_4 = \Lambda_3(\Re \to \Im)$$

$$\Lambda_5 = -\left|-\frac{4}{\sqrt{42}}|\Re(\bar{y})| + \frac{16\|\mathbf{h}\|^2}{42}\right| + \frac{8\|\mathbf{h}\|^2}{42}$$

$$\Lambda_6 = \Lambda_5(\Re \to \Im)$$

$$\Lambda_1 \approx \Re(\bar{y})$$

$$\Lambda_2 \approx \Im(\bar{y})$$

$$\Lambda_3 \approx -|\Re(\bar{y})| + \frac{4}{\sqrt{42}}\|\mathbf{h}\|^2$$

$$\Lambda_4 \approx -|\Im(\bar{y})| + \frac{4}{\sqrt{42}}\|\mathbf{h}\|^2$$

$$\Lambda_5 = -\left|-|\Re(\bar{y})| + \frac{4\|\mathbf{h}\|^2}{\sqrt{42}}\right| + \frac{2\|\mathbf{h}\|^2}{\sqrt{42}}$$

$$\Lambda_6 = -\left|-|\Im(\bar{y})| + \frac{4\|\mathbf{h}\|^2}{\sqrt{42}}\right| + \frac{2\|\mathbf{h}\|^2}{\sqrt{42}}$$

- Scale LLRs by $\frac{4}{\sqrt{42}}$ and approximate first 4 bits

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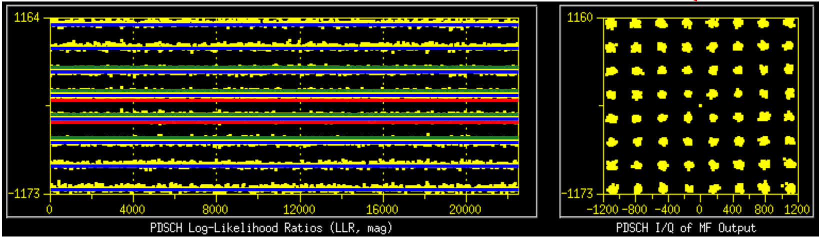
Simulation 64-QAM

$\Lambda_1 \approx \Re(\bar{y})$ $\Lambda_3 \approx -|\Re(\bar{y})| + \frac{4}{\sqrt{42}}\|\mathbf{h}\|^2$ $\Lambda_5 = -\left|-|\Re(\bar{y})| + \frac{4\|\mathbf{h}\|^2}{\sqrt{42}}\right| + \frac{2\|\mathbf{h}\|^2}{\sqrt{42}}$

$\Lambda_2 \approx \Im(\bar{y})$ $\Lambda_4 \approx -|\Im(\bar{y})| + \frac{4}{\sqrt{42}}\|\mathbf{h}\|^2$ $\Lambda_6 = -\left|-|\Im(\bar{y})| + \frac{4\|\mathbf{h}\|^2}{\sqrt{42}}\right| + \frac{2\|\mathbf{h}\|^2}{\sqrt{42}}$

$\bar{y} = \|\mathbf{h}\|^2 s + \tilde{n}$



- $\frac{7}{12}$ of LLRs are ± 1, $\frac{3}{12}$ are ± 3 and $\frac{1}{12}$ are ± 5 or ± 7

Outline

Multiple Streams to Single User

Transmission Model

- $\mathbf{y} = \bar{\mathbf{H}}\mathbf{s} + \mathbf{n} = \mathbf{H}\sum_{k=1}^{n_\mathrm{s}} \mathbf{g}_k s_k + \mathbf{n}$
- Effective channel $\bar{\mathbf{H}} = \mathbf{H}\mathbf{G}$, $\bar{\mathbf{h}}_k = \mathbf{H}\mathbf{g}_k$
- $\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\Omega_M^{n_\mathrm{s}}} \{\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2\}$

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Multiple Streams to Single User

Transmission Model

- $\mathbf{y} = \bar{\mathbf{H}}\mathbf{s} + \mathbf{n} = \mathbf{H}\sum_{k=1}^{n_\mathrm{s}} \mathbf{g}_k s_k + \mathbf{n}$
- Effective channel $\bar{\mathbf{H}} = \mathbf{H}\mathbf{G}$, $\bar{\mathbf{h}}_k = \mathbf{H}\mathbf{g}_k$
- $\hat{\mathbf{s}} = \arg\min_{\mathbf{s}\in\Omega_M^{n_\mathrm{s}}}\{\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2\}$

Distance Metric Computation

$$\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2 = \|\mathbf{y} - \sum_{k=1}^{n_\mathrm{s}} \bar{\mathbf{h}}_k s_k\|^2$$

$$= \|\mathbf{y}\|^2 + \sum_{k=1}^{n_\mathrm{s}} \|\bar{\mathbf{h}}_k\|^2 |s_k|^2 - 2\sum_{k=1}^{n_\mathrm{s}} \Re(\bar{y}_k s_k^*) + 2\sum_{k=1}^{n_\mathrm{s}}\sum_{j=k+1}^{n_\mathrm{s}} \Re(\rho_{kj} s_k^* s_j)$$

- MF $\bar{y}_k = \bar{\mathbf{h}}_k^\mathrm{H}\mathbf{y}$ and correlation factor $\bar{\rho}_{kj} = \bar{\mathbf{h}}_k^\mathrm{H}\bar{\mathbf{h}}_j$

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Special Case: 2×2 SU-MIMO

MaxLog Approximation $\lambda = \max\left\{-\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2\right\}$

$$\lambda = \max_{\mathbf{s}\in\mathcal{Q}_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\mathfrak{R}(\bar{y}_1)\mathfrak{R}(s_1) + \mathfrak{I}(\bar{y}_1)\mathfrak{I}(s_1)] - 2\eta_1\mathfrak{R}(s_2) - 2\eta_2\mathfrak{I}(s_2)\}$$

- $\eta_1 = \mathfrak{R}(\bar{\rho}_{12})\mathfrak{R}(s_1) + \mathfrak{I}(\bar{\rho}_{12})\mathfrak{I}(s_1) - \mathfrak{R}(\bar{y}_2)$ and
- $\eta_2 = \mathfrak{R}(\bar{\rho}_{12})\mathfrak{I}(s_1) - \mathfrak{I}(\rho_{12})\mathfrak{R}(s_1) - \mathfrak{I}(\bar{y}_2)$ are independent of s_2!

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Special Case: 2×2 SU-MIMO

MaxLog Approximation $\lambda = \max\left\{-\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2\right\}$

$$
\begin{aligned}
\lambda &= \max_{\mathbf{s}\in\Omega_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1)] - 2\eta_1\Re(s_2) - 2\eta_2\Im(s_2)\} \\
&= \max_{\mathbf{s}\in\Omega_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1)] + 2|\eta_1||\Re(s_2)| + 2|\eta_2||\Im(s_2)|\}
\end{aligned}
$$

- $\eta_1 = \Re(\bar{\rho}_{12})\Re(s_1) + \Im(\bar{\rho}_{12})\Im(s_1) - \Re(\bar{y}_2)$ and
- $\eta_2 = \Re(\bar{\rho}_{12})\Im(s_1) - \Im(\rho_{12})\Re(s_1) - \Im(\bar{y}_2)$ are independent of s_2!
- Maximized if $\Re(s_2)$, $\Im(s_2)$ have opposite sign of η_1,η_2, respectively $\Rightarrow -2\eta_1\Re(s_2) = 2|\eta_1||\Re(s_2)|$ and $-2\eta_2\Im(s_2) = 2|\eta_2||\Im(s_2)|$

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Special Case: 2×2 SU-MIMO

MaxLog Approximation $\lambda = \max\left\{-\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2\right\}$

$$\lambda = \max_{\mathbf{s} \in \Omega_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1)] - 2\eta_1\Re(s_2) - 2\eta_2\Im(s_2)\}$$

$$= \max_{\mathbf{s} \in \Omega_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1)] + 2|\eta_1||\Re(s_2)| + 2|\eta_2||\Im(s_2)|\}$$

- $\eta_1 = \Re(\bar{\rho}_{12})\Re(s_1) + \Im(\bar{\rho}_{12})\Im(s_1) - \Re(\bar{y}_2)$ and
- $\eta_2 = \Re(\bar{\rho}_{12})\Im(s_1) - \Im(\rho_{12})\Re(s_1) - \Im(\bar{y}_2)$ are independent of s_2!
- Maximized if $\Re(s_2)$, $\Im(s_2)$ have opposite sign of η_1,η_2, respectively $\Rightarrow -2\eta_1\Re(s_2) = 2|\eta_1||\Re(s_2)|$ and $-2\eta_2\Im(s_2) = 2|\eta_2||\Im(s_2)|$
- Setting the derivative w.r.t. $|\Re(s_2)|$, $|\Im(s_2)|$ to zero yields $|\Re(s_2)|^\star = \frac{|\eta_1|}{\|\bar{\mathbf{h}}_2\|^2}$ and $|\Im(s_2)|^\star = \frac{|\eta_2|}{\|\bar{\mathbf{h}}_2\|^2}$

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Special Case: 2×2 SU-MIMO

MaxLog Approximation $\lambda = \max\left\{-\|\mathbf{y} - \bar{\mathbf{H}}\mathbf{s}\|^2\right\}$

$$\lambda = \max_{\mathbf{s}\in\Omega_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1)] - 2\eta_1\Re(s_2) - 2\eta_2\Im(s_2)\}$$

$$= \max_{\mathbf{s}\in\Omega_M^{n_S}} \{-\|\bar{\mathbf{h}}_1\|^2|s_1|^2 - \|\bar{\mathbf{h}}_2\|^2|s_2|^2 + 2[\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1)] + 2|\eta_1||\Re(s_2)| + 2|\eta_2||\Im(s_2)|\}$$

- $\eta_1 = \Re(\bar{\rho}_{12})\Re(s_1) + \Im(\bar{\rho}_{12})\Im(s_1) - \Re(\bar{y}_2)$ and
- $\eta_2 = \Re(\bar{\rho}_{12})\Im(s_1) - \Im(\rho_{12})\Re(s_1) - \Im(\bar{y}_2)$ are independent of s_2!
- Maximized if $\Re(s_2)$, $\Im(s_2)$ have opposite sign of η_1,η_2, respectively $\Rightarrow -2\eta_1\Re(s_2) = 2|\eta_1||\Re(s_2)|$ and $-2\eta_2\Im(s_2) = 2|\eta_2||\Im(s_2)|$
- Setting the derivative w.r.t. $|\Re(s_2)|$, $|\Im(s_2)|$ to zero yields $|\Re(s_2)|^\star = \frac{|\eta_1|}{\|\bar{\mathbf{h}}_2\|^2}$ and $|\Im(s_2)|^\star = \frac{|\eta_2|}{\|\bar{\mathbf{h}}_2\|^2}$
- Search reduced by one complex dimension
- To detect s_2 exchange indexes $1, 2$

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QPSK-QPSK

Maximization takes the form

$$\lambda = \max_{\substack{s_1 \in \mathcal{Q}_4 \\ s_2 \in \mathcal{Q}_4}} \{\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1) + |\eta_1||\Re(s_2)| + |\eta_2||\Im(s_2)|\}$$

QPSK-QPSK

Maximization takes the form

$$\lambda = \max_{\substack{s_1 \in \mathcal{Q}_4 \\ s_2 \in \mathcal{Q}_4}} \{\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1) + |\eta_1||\Re(s_2)| + |\eta_2||\Im(s_2)|\}$$

$$\begin{aligned}
\Lambda_1(s_1) = \max & \left\{ \left| \frac{\Re(\bar{y}_2)}{2} - \frac{\bar{\rho}_{12}}{\sqrt{8}} \right| + \left| \frac{\Im(\bar{y}_2)}{2} - \frac{\bar{\rho}^*_{12}}{\sqrt{8}} \right| + \frac{\Im(\bar{y}_1)}{2}, \right. \\
& \left. \left| \frac{\Re(\bar{y}_2)}{2} - \frac{\bar{\rho}^*_{12}}{\sqrt{8}} \right| + \left| \frac{\Im(\bar{y}_2)}{2} + \frac{\bar{\rho}_{12}}{\sqrt{8}} \right| - \frac{\Im(\bar{y}_1)}{2} \right\} \\
- \max & \left\{ \left| \frac{\Re(\bar{y}_2)}{2} + \frac{\bar{\rho}^*_{12}}{\sqrt{8}} \right| + \left| \frac{\Im(\bar{y}_2)}{2} - \frac{\bar{\rho}_{12}}{\sqrt{8}} \right| + \frac{\Im(\bar{y}_1)}{2}, \right. \\
& \left. \left| \frac{\Re(\bar{y}_2)}{2} + \frac{\bar{\rho}_{12}}{\sqrt{8}} \right| + \left| \frac{\Im(\bar{y}_2)}{2} + \frac{\bar{\rho}^*_{12}}{\sqrt{8}} \right| - \frac{\Im(\bar{y}_1)}{2} \right\} + \Re(\bar{y}_1)
\end{aligned}$$

- With $\bar{\rho}_{12} = \Re(\bar{\rho}_{12}) + \Im(\bar{\rho}_{12})$ and $\bar{\rho}^*_{12} = \Re(\bar{\rho}_{12}) - \Im(\bar{\rho}_{12})$

QPSK-QPSK

Maximization takes the form

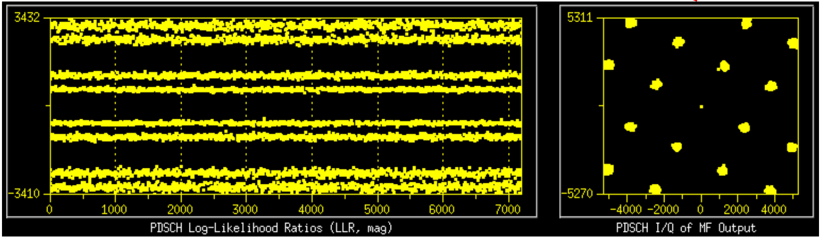
$$\lambda = \max_{\substack{s_1 \in \mathcal{Q}_4 \\ s_2 \in \mathcal{Q}_4}} \{\Re(\bar{y}_1)\Re(s_1) + \Im(\bar{y}_1)\Im(s_1) + |\eta_1||\Re(s_2)| + |\eta_2||\Im(s_2)|\}$$

$$\begin{aligned}
\Lambda_2(s_1) = \max\Bigg\{ & \left|\frac{\Re(\bar{y}_2)}{2} - \frac{\bar{\rho}_{12}}{\sqrt{8}}\right| + \left|\frac{\Im(\bar{y}_2)}{2} - \frac{\bar{\rho}_{12}^*}{\sqrt{8}}\right| + \frac{\Re(\bar{y}_1)}{2}, \\
& \left|\frac{\Re(\bar{y}_2)}{2} + \frac{\bar{\rho}_{12}^*}{\sqrt{8}}\right| + \left|\frac{\Im(\bar{y}_2)}{2} - \frac{\bar{\rho}_{12}}{\sqrt{8}}\right| - \frac{\Re(\bar{y}_1)}{2} \Bigg\} \\
- \max\Bigg\{ & \left|\frac{\Re(\bar{y}_2)}{2} - \frac{\bar{\rho}_{12}^*}{\sqrt{8}}\right| + \left|\frac{\Im(\bar{y}_2)}{2} + \frac{\bar{\rho}_{12}}{\sqrt{8}}\right| + \frac{\Re(\bar{y}_1)}{2}, \\
& \left|\frac{\Re(\bar{y}_2)}{2} + \frac{\bar{\rho}_{12}}{\sqrt{8}}\right| + \left|\frac{\Im(\bar{y}_2)}{2} + \frac{\bar{\rho}_{12}^*}{\sqrt{8}}\right| - \frac{\Re(\bar{y}_1)}{2} \Bigg\} + \Im(\bar{y}_1).
\end{aligned}$$

- With $\bar{\rho}_{12} = \Re(\bar{\rho}_{12}) + \Im(\bar{\rho}_{12})$ and $\bar{\rho}_{12}^* = \Re(\bar{\rho}_{12}) - \Im(\bar{\rho}_{12})$

Simulation QPSK-QPSK

$$\bar{y}_1 = \|\bar{\mathbf{h}}_1\|^2 s_1 + \bar{\rho}_{12} s_2 + \tilde{n}$$



MU-MISO

Transmission Model

- $y_k = \bar{\mathbf{h}} s_k + \sum_{j \neq k}^{n_s} \bar{\mathbf{h}}_j s_j + n_k$
- Effective channel $\bar{\mathbf{h}}_i = \mathbf{h}_k \mathbf{g}_i$

Comparison to SU-MIMO

- Transmission model identical except no need to decode interfering symbol
- Interfering symbol constellation unknown

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Inter-cell Interference

Transmission Model

- $\mathbf{y}_k = \mathbf{H}_k \mathbf{s}_k + \sum_{j=1, j \neq k}^{K} \mathbf{H}_j \mathbf{s}_j + \mathbf{n}_k$

Challenges

- UE at cell edge syncs to interfering cell $\Rightarrow$ Chest
- Pilots and data of interfering cell and serving cell interfere $\Rightarrow$ bad Chest
- DLSCH power control (ρ_A, ρ_B) can be used to decrease pilot to data power
- Useful in scenarios where UE could decode at low SNR but is strongly interfered
- How good must Chest be for IA to work?

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Outline

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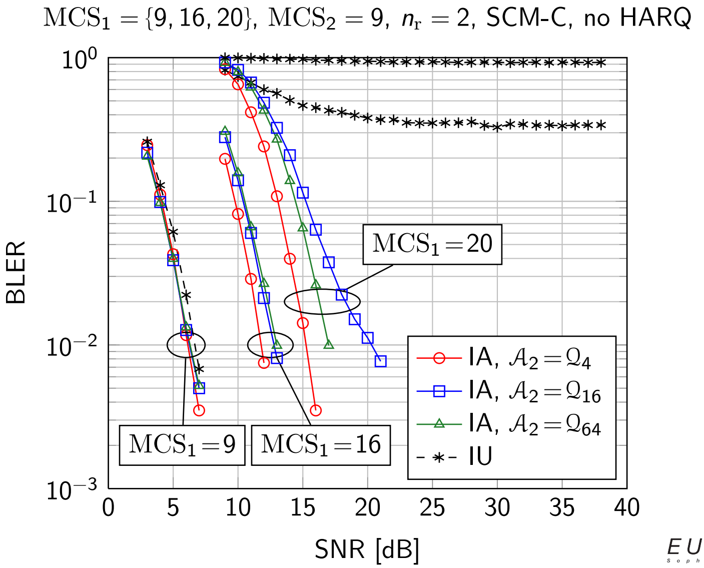
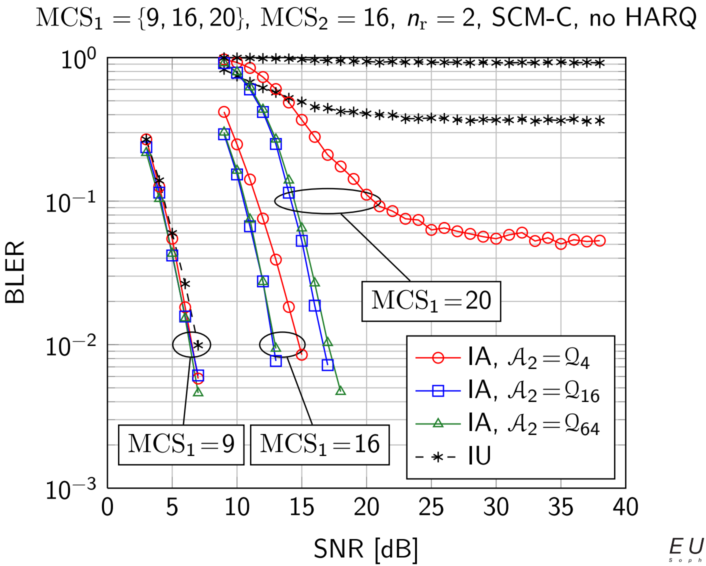
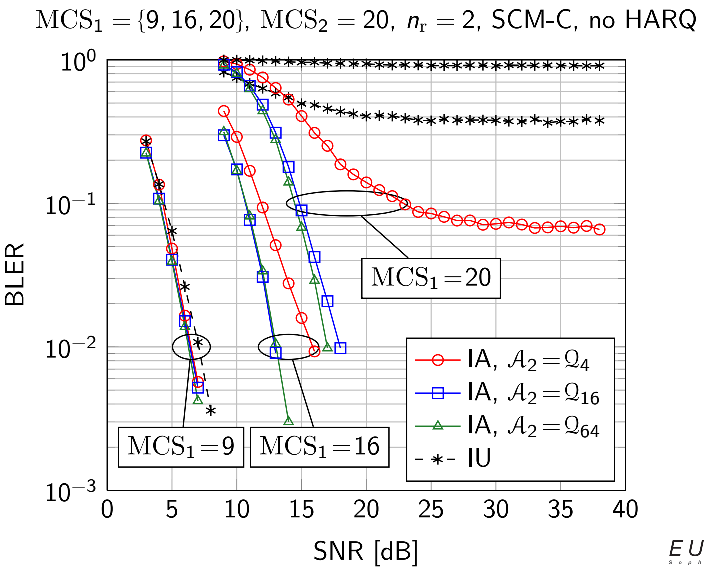
QPSK Interference



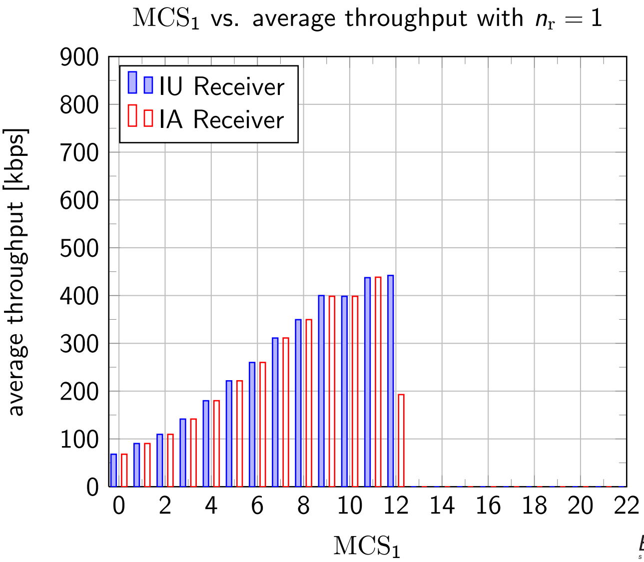
16QAM Interference



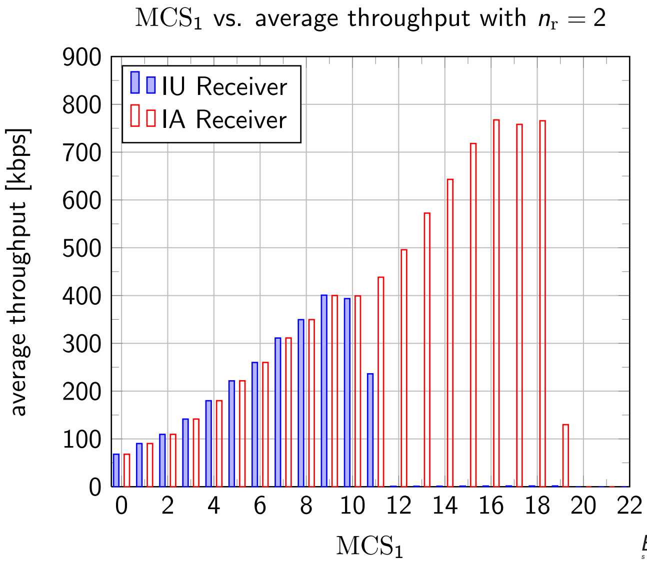
64QAM Interference



Single-Antenna Receiver



Dual-Antenna Receiver



Outline

Summary

- IA receiver achieves significant gains even with false assumption on interfering symbol constellation
- Throughput increases drastically if multiple receive antennas are available
- If interfering constellation unknown, assume the same as desired constellation
- Extension of IA receiver to different interference scenarios possible
- To employ the IA receiver requires knowledge that interferer is actually present
- How well does IA receiver work for bad channel estimate of interferer?

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